

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Manipulating Polynomials

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

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Manipulating Polynomials

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to manipulate and calculate with polynomials. In particular, it aims to identify and help students who have difficulties in:

- Switching between visual and algebraic representations of polynomial expressions.
- Performing arithmetic operations on algebraic representations of polynomials, factorizing and expanding appropriately when it helps to make the operations easier.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

- A-APR: Perform arithmetic operations on polynomials
- A-SSE: Interpret the structure of expressions

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

2. Reason abstractly and quantitatively.
7. Look for and make use of structure.
8. Look for and make use of repeated reasoning.

INTRODUCTION

In this lesson students make connections between representations, clear parentheses, simplify, factorize, and use the arithmetic of polynomials in a purposive context.

- Before the lesson students work individually on the assessment task *Sequences and Equations*. You review their responses and create questions for students to consider, to help improve their work.
- You introduce the lesson by showing students how the card matching activity works. Students then work collaboratively in small groups to match diagrammatic and algebraic representations of polynomials. They display their work on posters.
- Students share their finished posters, comparing reasoning, and checking that explanations are clear and complete.
- In a whole-class discussion, you reflect on the strategies students have used to complete the task.
- Students then work individually, to review their solutions to the initial assessment task before completing a second task, *Sequences and Equations (revisited)*.

MATERIALS REQUIRED

- Each student will need a copy of the assessment tasks, *Sequences and Equations* and *Sequences and Equations (revisited)*, a mini-whiteboard, pen, and wipe.
- Each small group of students will need *Card Set: Sequences of Dots*, *Card Set: Adding Polynomials*, a large sheet of poster paper, a glue stick and a pencil. We suggest that before the lesson you cut-up the card sets.
- Students requiring an extension task should be given *Card Set: Blank Cards* so that they can create their own sequences of dots with algebraic expressions.
- There is a projector resource to support the group work and whole-class discussions.

TIME NEEDED

15 minutes before the lesson, a 65-minute lesson, and 15 minutes in a follow-up lesson (or for homework). These timings are approximate. Exact timings will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: Sequences and Equations (15 minutes)

Have the students do this task, in class or for homework, a day or more before the lesson. This will give you an opportunity to assess their work, and to find out the kinds of difficulties students have with the task. You should then be able to target your help more effectively in the follow-up lesson. Give each student a copy of the task.

Introduce the task briefly, and explain the purpose of the task.

This task is about sequences of patterns and algebraic expressions and shows how you can use diagrams to help understand the algebra.

Do not worry if you cannot answer all the questions. Next lesson we will work on a task that should help you progress on this topic.

Spend 15 minutes, working alone.

It is important that, as far as possible, students answer the questions without assistance. If students are struggling to get started, ask questions that help them to understand what they are being asked to do. The first few questions on the *Common issues* table (on the next page) may be helpful.

Sequences and Equations

1. These dot diagrams show the beginning of a sequence of patterns:

$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$

(a) Draw the fifth pattern in the sequence in the space above.

(b) How many dots of each color will there be in the 10th pattern? White: Black:

(c) The number of black dots in the n th diagram is given by the expression: $n + 1$.
Write algebraic expressions for the number of white dots and for the total number of dots:

Number of white dots	Number of black dots	Total number of dots
.....	$n + 1$
+		=

2. These dot diagrams show the beginning of another sequence of patterns:

$n = 1$	$n = 2$	$n = 3$	$n = 4$

(a) Complete the equation below with algebraic expressions for the number of white dots, number of black dots, and total number of dots:

Number of white dots	Number of black dots	Total number of dots
.....
+		=

(b) Rewrite the expression for the total in its simplest, factorized form if you haven't done so already.

.....

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding, and any difficulties they encountered.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare scores and distract their attention from what they might do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given in the *Common issues* table on the next page.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions, and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students, and write these on the board when you return the work to the students.

Common issues:**Suggested questions and prompts:**

<p>Student draws dot diagram incorrectly</p> <p>For example: The student draws the correct number of dots, but does not follow the pattern.</p> <p>Or: The student does not see the pattern at all and draws a dot diagram containing the too many/few dots.</p>	<ul style="list-style-type: none"> Describe how the dot diagram changes as n increases. What is happening to the number of white dots? What is happening to the number of black dots? What stays the same when you move from one diagram to the next? What changes? In what way does it change? Can you see a pattern in the way the dots are arranged?
<p>Student relies on counting</p> <p>For example: The student draws diagrams for $n = 6, 7, 8, 9$, and then counts the number of dots (Q1b.)</p>	<ul style="list-style-type: none"> How could you work out the number of dots in the 10th pattern without counting them all? Is this the most efficient method? How could you find the number of dots without drawing diagrams 6,7, 8, and 9?
<p>Student cannot link the dot diagrams to the algebra</p> <p>For example: The student just counts the dots and works out the number sequence.</p>	<ul style="list-style-type: none"> How does the algebraic expression '$n + 1$' connect to the pattern of black dots? Can you draw on the diagram to show this? Which parts of the pattern always contain n dots?
<p>Student provides numbers of dots for a specific case rather than an algebraic expression</p> <p>For example: The student writes $9 + 16 = 25$ as the solution to Q2a.</p>	<ul style="list-style-type: none"> You have identified the number of dots in the 4th pattern. How could you describe the number of dots in terms of n? Can you see a pattern in the way the dots are arranged? What shape are the dots arranged in?
<p>Student's factorization of algebraic expressions is incorrect or omitted</p> <p>For example: In Q2b. the student leaves the algebraic expression for the total number of dots as $(n - 1)^2 + 4n$ or similar.</p>	<ul style="list-style-type: none"> How could you combine these two expressions into one? What do you know about $(n - 1)^2$? Look at the dot diagrams as a whole (black and white dots together). What do you see? How could you use what you notice to write an algebraic expression for the total number of dots?
<p>Student completes the task</p>	<ul style="list-style-type: none"> Can you find two different algebraic expressions that add to give $3n$ or $(n + 1)^2$? What would the dot diagrams look like for these expressions?

SUGGESTED LESSON OUTLINE

Introduction: Dot Diagrams (10 minutes)

Introduce the lesson:

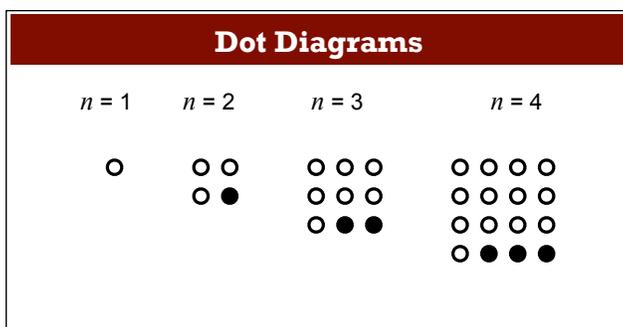
At the end of this lesson, I'm going to ask you to review your work on Sequences and Equations. First, we are going to do some group work on connecting dot diagrams to polynomials.

Check that the class are not confused by the term 'polynomial'; there is no need to go into great depth.

Do you remember what a polynomial is? [Linear expressions, quadratics, cubics, but not things like $1/x$ or square roots.]

The algebraic expressions $4n$, $n^2 + 1$ are examples of polynomials.

Give each student a mini-whiteboard, pen, and wipe. Display Slide P-1 of the projector resource:



Take time to check that students understand the use of dot diagrams to represent a sequence of patterns.

Describe the pattern you see in the dots.

Students may well talk about the square pattern of the whole array and ignore the different color dots.

On your whiteboard draw the diagram for $n = 5$ []

Once students have had a chance to draw a diagram for $n = 5$, display Slide P-2 and discuss any variations in the diagrams produced. Once students are all agreed on what the diagram looks like, discuss its features:

What can you tell me about the number of black dots? [e.g. they increase by one each time]

What can you tell me about the number of white dots? [e.g. they increase by 2, 4, 6, 8...]

The pattern for the sequence of white dots is less obvious. Encourage students to look at groups of dots. For example, the $n = 4$ diagram has a group of 4 by 3 dots plus 1 dot, the $n = 3$ diagram has a group of 3 by 2 dots plus 1 dot, etc. Students may find it helpful to draw around groups of dots.

How many black dots would there be in the n th diagram? [$n - 1$]

How many white dots would there be in the n th diagram? [$n^2 - n + 1$]

Allow students to explore different ways of visualizing the dot groupings. For example, some students may describe the white dots with the algebraic expression " $n(n - 1) + 1$ " or " $(n - 1)^2 + n$ " or " $n^2 - (n - 1)$ ".

Once students have had sufficient time to consider the patterns of white and black dots, reinforce the idea that the black and white dots add together to give the complete dot diagram. Give students the opportunity to discuss and complete the table on Slide P-3 of the projector resource. The table may be completed with, for example, the following algebraic expressions:

Number of white dots		Number of black dots		Total number of dots
$(n-1)^2 + n$	+	$n-1$	=	n^2

Explain to students that during this lesson they are going to be working with some similar diagrams and tables, as well as manipulating the algebraic expressions to check that the polynomial equations work.

Small group work: Matching Representations (30 minutes)

Organize students into groups of two or three.

Give each group cut-up copies of *Card Set: Sequence of Dots* (three sheets) and *Card Set: Adding Polynomials*, a large sheet of poster paper, and a glue stick. They will also need a pencil.

Students are to work in their small groups, matching sequences of dot diagrams to tables of algebraic expressions, generated by adding polynomials.

Introduce the lesson activity:

You are going to use these cards to make a poster that shows the connections between dot diagrams and algebraic expressions.

Start by choosing a dot diagram card and look for a polynomial card that matches it. You may need to complete some of the missing information to be able to do this.

See if you can spot patterns in the dot diagrams that match the algebra, rather than counting all the dots. You may find it helpful to draw around groups of dots.

Explain how students should work together:

Take turns to find a match. When it's your turn, explain your reasoning clearly.

The others in the group should listen and watch.

If they agree with the reasons given, they should explain again in their own words. If they disagree, they must explain their reasons. Continue to debate until you have all come to an agreement.

It may be that the card has been correctly matched but each member of the group describes the number of white/black dots with a different algebraic expression. Check to see if these expressions are equivalent.

Once you have happy with the matching, move on to the next card.

These instructions are summarized on Slide P-4 of the projector resource.

The purpose of this structured work is to encourage each student to engage with their partner's explanations and to take responsibility for their partner's understanding. Students should use their mini-whiteboards to share ideas and explain their thinking to each other.

During small group work, you have two tasks, to notice students' approaches and difficulties and to support them working as a group:

Notice students' approaches and difficulties

Observe students as they work together on the task. Are students finding correct matches? Do they recognize and draw structure in dot diagrams, and link this to the algebra? Do they manipulate the algebraic expressions into different forms by expanding, factorizing, and collecting like terms? Do students within a group use the same strategies for matching the cards? Are they substituting numbers into expressions and counting dots? Are they starting with the dot diagrams and writing down the structures they see, or looking at the algebra first? Do they reveal any common misconceptions, such as $(n + 1)^2 = n^2 + 1$?

You may want to use the questions in the *Common issues* table to help address misconceptions.

Support students working as a group

Support students in working together, encouraging them to take turns. If you notice that only one partner is matching/completing cards, ask the other student(s) to explain the reasons for the match/algebraic expressions.

Amy matched these two cards. Linda, why do you think these two cards go together?

If students in the group take different approaches when matching cards, encourage them to clearly explain the basis for a choice.

Why have you matched these two cards? Can anyone explain this match in a different way?

Can you explain why the number of white/black dots can be represented by this expression? How else could you explain this?

Try to avoid giving students the information they need to match pairs. Instead, focus on helping them develop strategies for working, such as leaving cards they cannot match until last, and looking for differences and similarities. Ask them to recall different ways of manipulating algebraic expressions such as clearing parentheses and factorizing, but do not give them hints about which techniques to use. Encourage students to divide up the dots in the diagrams in patterned ways, asking them to explain how to change from one diagram to the next, and how the structures match those in the polynomials.

Can you show the expression for the number of white/black dots on the diagram? How else could you explain this?

If one group struggles with an element of the task, you could refer them to a group experiencing more success. If a group struggles for more than 5 minutes to make any matches, advise them to concentrate on the dot diagrams. Can they write an algebraic expression for the number of white dots? Can they write an algebraic expression for the number of black dots? They may find it helpful to draw the next diagram in the sequence, to focus their attention on the structure of the diagrams.

As groups begin to finalize matches, make sure that they have completed the polynomial cards with algebraic expressions.

If you are happy with your matches, check that you have completed the algebraic expressions on the polynomials cards.

You now need to glue down the cards and add an explanation of why these cards match. Include on your poster any equivalent algebraic expressions you have used to help you to match the cards.

Make sure you spend some time carefully checking the algebra. You may need to clear parentheses or factorize as you do this.

Whilst students work on their posters, observe and support them as before. Some groups may not manage to match all of the cards and it is not essential that they do so. If students do successfully complete all of the matches, give them a copy of *Card Set: Blank Cards*. Ask them to design two new cards of their own, which they can then ask someone else in the group to complete. They will need to draw the dot diagrams for their chosen sequence as well as deciding which algebraic expressions to include.

Sharing posters (10 minutes)

Once students have finished their posters, get them to share their work.

Now, one person from each group, get up and visit a different group and look carefully at their poster. Do you understand their explanations? Ask them to explain anything you don't understand.

After a few minutes, get students to reverse roles: the visitors should return to their own posters and their partners should visit another group. It does not matter if students visit a poster that contains different matched cards to the ones on their poster. The aim of this activity is to give students the opportunity to critique each other's explanations.

Whole-class discussion (15 minutes)

Organize a whole-class discussion to explore the different strategies used by students when matching the cards, as well as identifying areas in which students struggled. It is likely that some groups will not have matched all of the cards and so the aim of this discussion is to encourage contributions from everyone in the class, rather than focusing on specific card matches.

Give me a pair of cards that was easy to match. Why was this?

Did anyone find these cards difficult to match or use a different strategy to the one explained?

Encourage students to describe why their expressions and equations match the dot diagram, rather than merely describing the process of matching the cards.

Why does this expression represent the number of white/black dots? Can you show it on the dot diagram?

How else could you explain it?

Did anyone write down a different expression for the number of white/black dots?

Students may well assume that an expression is different, when in fact it is the same expression written in a different form. Encourage students to check their algebra and manipulate their expressions when comparing their work during this whole-class discussion.

Which expression is the easiest to visualize/explain? Why is this?

How did you check the algebra when completing the polynomial equations?

How could this expression be simplified? Does simplifying make it easier to see the match or more difficult? Explain why this is.

Ask students what they learned by looking at other students' work and whether or not this helped them with dot diagrams that they had found difficult to match or were unsure about:

Which sequence of dots did you find most difficult to match to a polynomial? Why was this?

Had the group that you visited matched this sequence of dots to a polynomial? In what ways did their explanation of the match help?

Did anyone disagree with a match that another group had made? Explain why this was.

Try to draw out the strategies students used when matching the cards.

Did you notice any similarities in the cards, which helped you with subsequent matches?

Did you make any assumptions that resulted in incorrect matches? How did you recognize this?

During the discussion, address any issues you have noticed as students worked on the activity, making specific reference to any misconceptions. You may want to use the questions in the *Common issues* table to support this discussion.

Follow-up lesson: Reviewing the assessment task (15 minutes)

Give each student a copy of the assessment task *Sequences and Equations (revisited)*, along with their responses to the assessment task *Sequences and Equations* with your questions written on them. If you have not written questions on each script, write your list of questions on the board.

Read through your solutions to Sequences and Equations and think about the questions I have asked.

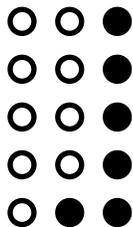
Now take a look at the second sheet: Sequences and Equations (revisited). Can you use what you have learned to answer these questions?

This task could be set for homework.

SOLUTIONS

Assessment task: Sequences and Equations

- 1 (a) Diagram for the fifth pattern:



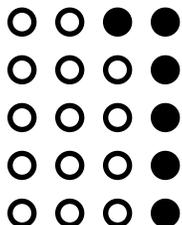
- (b) There will be 19 white dots and 11 black dots in the 10th pattern.

- (c) Number of white dots = $2n - 1$ and total number of dots = $3n$.

- 2 (a) & (b) Number of white dots = $(n - 1)^2$, number of black dots = $4n$ and total number of dots in simplest, factorized form = $(n + 1)^2$.

Assessment task: Sequences and Equations (revisited)

- 1 (a) Diagram for the fifth pattern:



- (b) Number of white dots = $3n - 1$ and total number of dots = $4n$.

- 2 (a) Number of white dots = $4n$, number of black dots = $n^2 + 4$ and total number of dots in simplest, factorized form = $(n + 2)^2$.

Lesson task:

A	$n = 1$	$n = 2$	$n = 3$	$n = 4$
B	$n = 1$	$n = 2$	$n = 3$	$n = 4$
C	$n = 1$	$n = 2$	$n = 3$	$n = 4$

5	White $2n$	+	Black $n^2 + 1$	=	Total $(n + 1)^2$
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6	White $n(n - 1)$	+	Black n	=	Total n^2
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7	White $(n - 1)^2$	+	Black $4n$	=	Total $(n + 1)^2$
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D	$n = 1$	$n = 2$	$n = 3$	$n = 4$

E	$n = 1$	$n = 2$	$n = 3$	$n = 4$

F	$n = 1$	$n = 2$	$n = 3$	$n = 4$

G	$n = 1$	$n = 2$	$n = 3$	$n = 4$

H	$n = 1$	$n = 2$	$n = 3$	$n = 4$

I	$n = 1$	$n = 2$	$n = 3$	$n = 4$

3	White $2n - 1$	+	Black $(n - 1)^2$	=	Total n^2
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1	White $2n - 1$	+	Black $2n + 1$	=	Total $4n$
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4	White $n + 1$	+	Black $n(n - 1)$	=	Total $n^2 + 1$
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8	White 1	+	Black $4n(n - 1)$	=	Total $(2n - 1)^2$
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2	White $\frac{n(n+1)}{2}$	+	Black $\frac{n(n+1)}{2}$	=	Total $n(n+1)$
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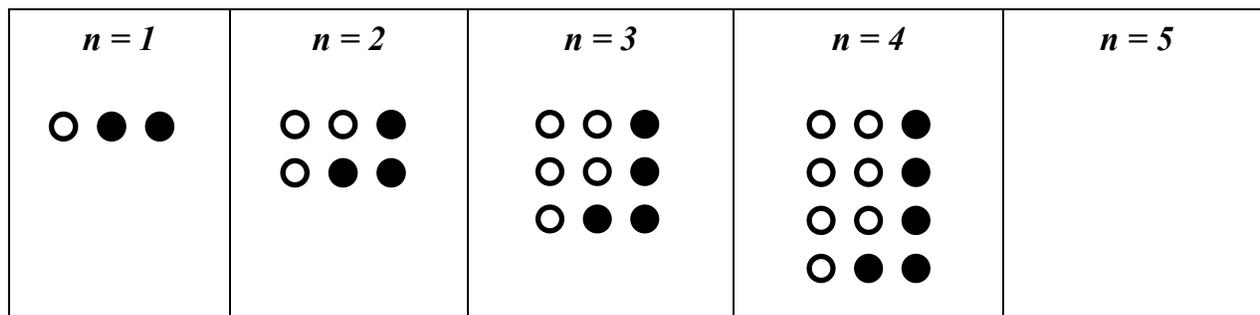
9	White $\frac{n(n+1)}{2}$	+	Black $\frac{n(n-1)}{2}$	=	Total n^2
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Students may well complete the polynomial cards with different, yet equivalent algebraic expressions to the ones shown above. For example, they may write $n + (n - 1)$ on Card 1 instead of $2n - 1$.

When students are making their poster, encourage them to simplify their algebra as part of the checking process.

Sequences and Equations

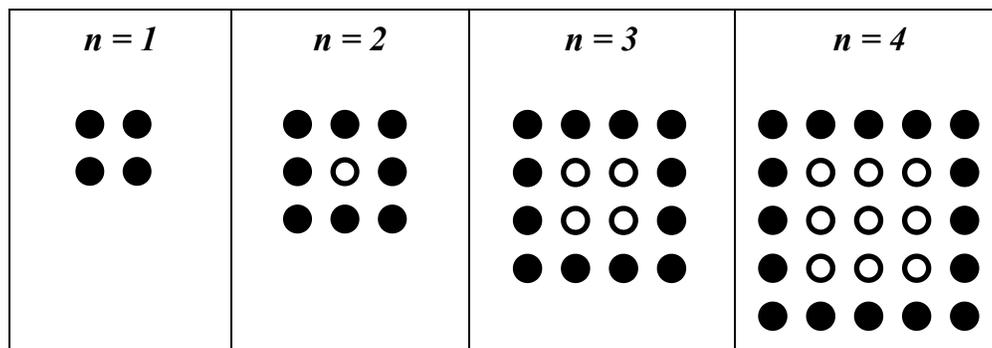
1. These dot diagrams show the beginning of a sequence of patterns:



- (a) Draw the fifth pattern in the sequence in the space above.
- (b) How many dots of each color will there be in the 10th pattern? White: Black:
- (c) The number of black dots in the n th diagram is given by the expression: $n + 1$.
Write algebraic expressions for the number of white dots and for the total number of dots:

Number of white dots	Number of black dots	Total number of dots
.....	$+ \quad n + 1$	$= \quad$

2. These dot diagrams show the beginning of another sequence of patterns:



- (a) Complete the equation below with algebraic expressions for the number of white dots, number of black dots, and total number of dots:

Number of white dots	Number of black dots	Total number of dots
.....	$+ \quad$	$= \quad$

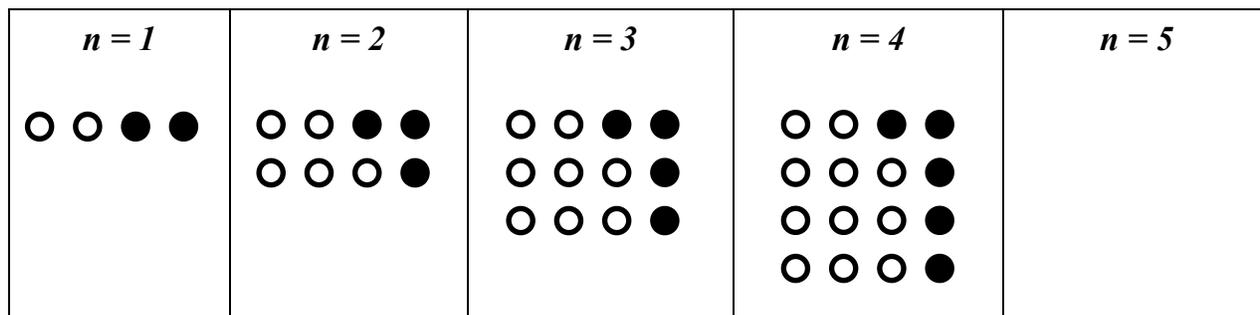
- (b) Rewrite the expression for the total in its simplest, factorized form, if you haven't done so already.

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Sequences and Equations (revisited)

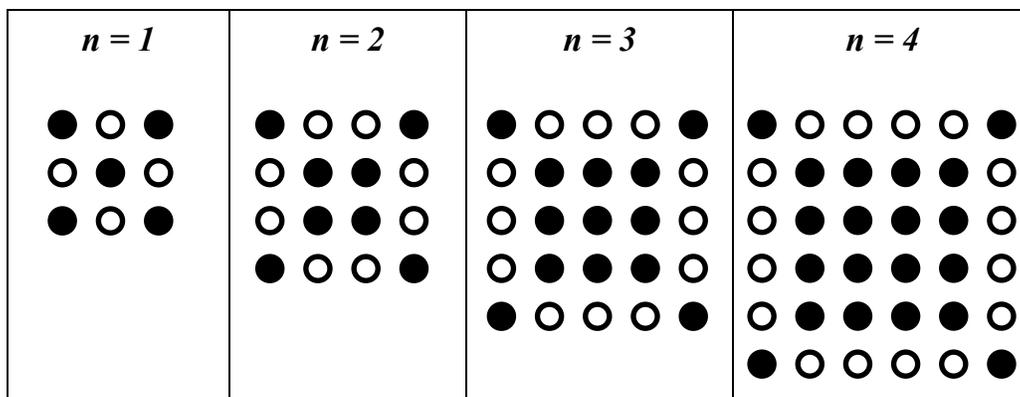
1. These dot diagrams show the beginning of a sequence of patterns:



- (a) Draw the fifth pattern in the sequence in the space above.
- (b) The number of black dots in the n th diagram is given by the expression: $n + 1$.
Write algebraic expressions for the number of white dots and for the total number of dots:

Number of white dots	Number of black dots	Total number of dots
.....	$+ \quad n + 1$	$= \quad$

2. These dot diagrams show the beginning of another sequence of patterns:



- (a) Complete the equation below with algebraic expressions for the number of white dots, number of black dots, and total number of dots:

Number of white dots	Number of black dots	Total number of dots
.....	$+ \quad$	$= \quad$

- (b) Rewrite the expression for the total in its simplest, factorized form, if you haven't done so already.

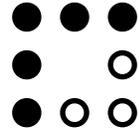
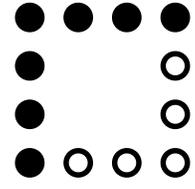
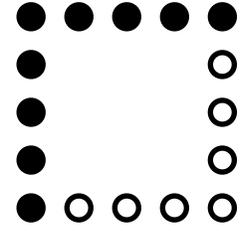
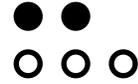
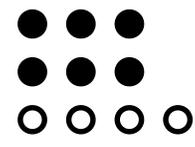
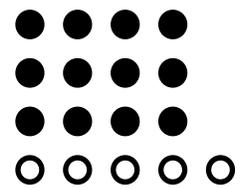
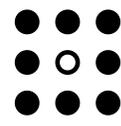
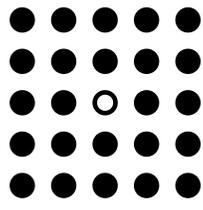
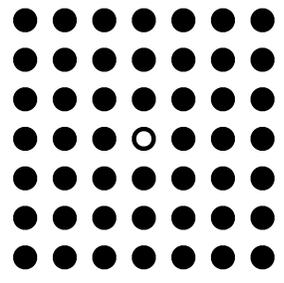
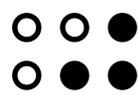
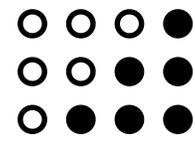
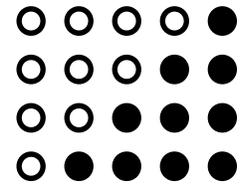
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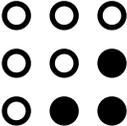
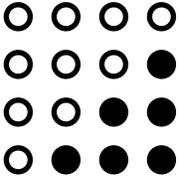
Card Set: Sequences of Dots

A	$n = 1$	$n = 2$	$n = 3$	$n = 4$
B	$n = 1$	$n = 2$	$n = 3$	$n = 4$
C	$n = 1$	$n = 2$	$n = 3$	$n = 4$
D	$n = 1$	$n = 2$	$n = 3$	$n = 4$

Card Set: Sequences of Dots (continued 1)

E	$n = 1$ 	$n = 2$ 	$n = 3$ 	$n = 4$ 
F	$n = 1$ 	$n = 2$ 	$n = 3$ 	$n = 4$ 
G	$n = 1$ 	$n = 2$ 	$n = 3$ 	$n = 4$ 
H	$n = 1$ 	$n = 2$ 	$n = 3$ 	$n = 4$ 

Card Set: Sequences of Dots (continued 2)

1	$n = 1$	$n = 2$	$n = 3$	$n = 4$
				

Card Set: Adding Polynomials

1	White	+	Black	=	Total $4n$
2	White	+	Black	=	Total $n(n+1)$
3	White	+	Black	=	Total n^2
4	White	+	Black	=	Total $n^2 + 1$
5	White	+	Black	=	Total $(n+1)^2$
6	White $n(n-1)$	+	Black	=	Total
7	White	+	Black $4n$	=	Total
8	White	+	Black	=	Total $(2n-1)^2$
9	White	+	Black $\frac{n(n-1)}{2}$	=	Total n^2

Card Set: Blank Cards

Now make up two sequences of your own:

$n = 1$	$n = 2$	$n = 3$	$n = 4$
$n = 1$	$n = 2$	$n = 3$	$n = 4$

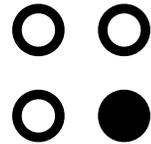
White		Black		Total
	+		=	
White		Black		Total
	+		=	

Dot Diagrams

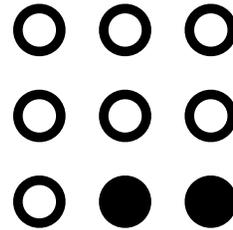
$n = 1$



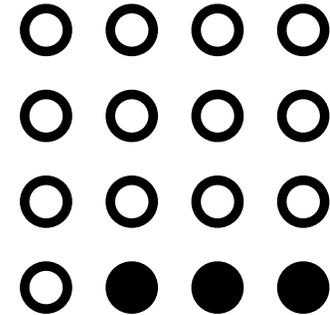
$n = 2$



$n = 3$



$n = 4$

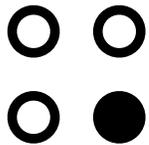


Dot Diagrams

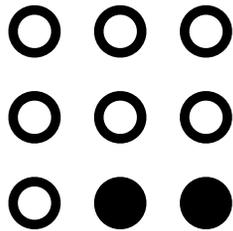
$n = 1$



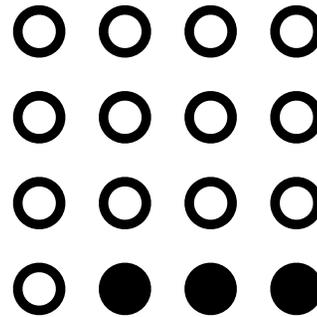
$n = 2$



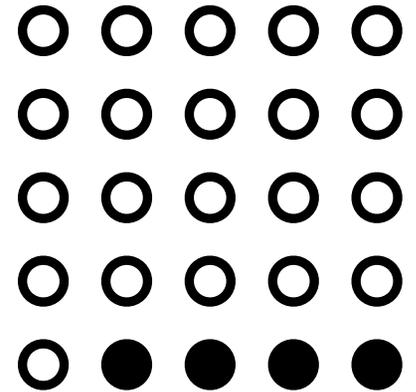
$n = 3$



$n = 4$



$n = 5$

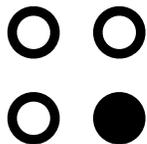


Dot Diagrams

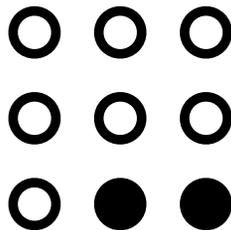
$n = 1$



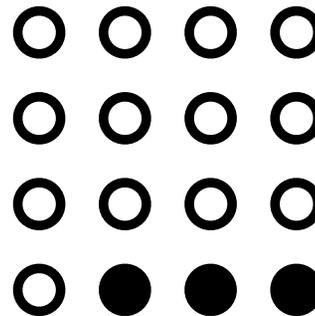
$n = 2$



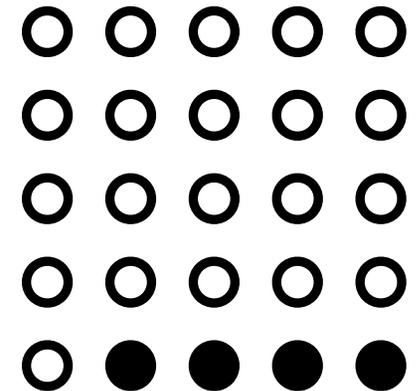
$n = 3$



$n = 4$



$n = 5$



Number of white dots		Number of black dots		Total number of dots
.....	+	=

Working in Groups

1. Take turns to find a match.
2. When it's your turn, explain your reasoning clearly.
3. If others in your group agree with your reasoning, they should explain it again in their own words. If they disagree, they must explain why they think you are incorrect.
4. Continue to debate until you have all come to an agreement.
5. Once you are happy with the matching, move onto the next card.

Mathematics Assessment Project
CLASSROOM CHALLENGES

This lesson was designed and developed by the
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at the
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It was refined on the basis of reports from teams of observers led by
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based on their observation of trials in US classrooms
along with comments from teachers and other users.

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